# Curve Sketching

**Objectives**

1. To learn how to use the first and second derivatives to find a good sketch of the graph of a function $f$.
2. To identify the points needed to frame the graph of a function $f$.
3. To learn how to determine the horizontal and the vertical asymptotes for a function.
4. To learn how to use infinite limits to locate asymptotes.

## Steps to Finding a Complete Graph of a Function *f*

1. Find $f'$ and $f''$.
2. Find all critical values of $f$.
3. Find intervals of increase for $f$ and intervals of decrease for $f$.
4. Find all critical values of $f'$.
5. Find intervals of upward concavity for $f$ and intervals of downward concavity for $f$.
6. Find all inflection points for $f$ from values found in step 4.
7. Locate all relative extreme values for $f$.
8. **Frame the graph**, i.e., plot on a coordinate system the points corresponding to values found in steps 2 and 4.
9. Use the frame to either sketch a complete graph of $f$ or set the viewing rectangle for a complete graph of $f$.

**Problem 25.1**

Find the following for the function $f(x) = x^3 - 27x$. Then sketch the graph.

Critical values of $f$ :

Intervals of Increase: Intervals of Decrease:

Critical values of $f'$:

Intervals of Concave Up: Intervals of Concave Down:

Inflection points:

Relative extreme points:

**Problem 25.2** movie

Find the following for the function $g(x) = x^4 + 4x^3 - 16x + 2$. Then sketch the graph.

Critical values of $f$:

Intervals of Increase:

Intervals of Decrease:

Critical values of $f'$:

Intervals of Concave Up:

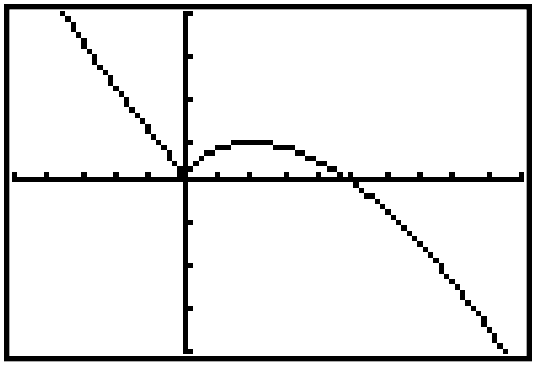
Intervals of Concave Down:

Inflection points:

Relative extreme points:

**Problem 25.3** flash movie

The graph of the function $g(t) = 5t^{2/3} - t^{5/3}$ in the viewing window [–5, 10] by [–20, 20] is given below.



(a) Use calculus to find all critical points for the function $g(x)$ and locate these critical points, giving their coordinates, on the viewing window above.

(b) Are there any points of inflection for the graph of $g(x)$ in the viewing window above? Use calculus to answer this question.

## Vertical Asymptotes

The line $x = b$ is called a **vertical asymptote** of the graph of $y = f(x)$ if at least one of the following statements is true:

$\lim_{x \to b} f(x) = +\infty$ $\quad$ $\lim_{x \to b^+} f(x) = +\infty$ $\quad$ $\lim_{x \to b^-} f(x) = +\infty$

$\lim_{x \to b} f(x) = -\infty$ $\quad$ $\lim_{x \to b^+} f(x) = -\infty$ $\quad$ $\lim_{x \to b^-} f(x) = -\infty$.

### Locating Vertical Asymptotes in Rational Functions

In the special case when $R(x)$ is a rational function, i. e., $R(x) = \dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials, then vertical asymptotes usually occur at the values $b$ such that $Q(b) = 0$.

## Horizontal Asymptotes

The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to +\infty} f(x) = L \text{ or } \lim_{x \to -\infty} f(x) = L.$$

**Note**: It is possible that a function $f$ may have two distinct horizontal asymptotes. This happens when $\lim_{x \to +\infty} f(x) = L_1$ and $\lim_{x \to -\infty} f(x) = L_2$ and $L_1 \neq L_2$.

### Problem 25.4

Find all asymptotes for each of the following functions. Then graph each function on your graphing calculator in an appropriate viewing window to examine the graphing behavior of the function near each of its asymptotes.

(a) $f(x) = \dfrac{x^2 + 2}{x^2 - 2}$;

(b) $g(x) = \dfrac{x^2 - 2}{x^2 + 2}$;

(c) $H(x) = \dfrac{\sqrt{2x^2 + 1}}{3x - 5}$.

**Problem 25.5 (P&V #9, p. 207)**

Sketch the graph of $f(x) = \dfrac{x^2}{x^2 - 4}$.

**Assignment for Lesson 25 (Purcell & Varberg, 7th Ed.)**

1. Complete all problems in LG/W Lesson 25
2. Read Section 4.8
3. Do the following problems.

   Page 217: 3, 7, 11

   Page 227: 23, 25, 27
4. Preview LG/W Lesson 26 and P & V Section 3.10

**Assignment for Lesson 25 (Stewart)**

1. Complete all problems in LG/W Lesson 25
2. Read Section 4.4
3. Do the following problems.

   Page 300: 1, 3, 5, 7, 11

   Also find all critical values and points of inflection for each of the functions in these problems using first and second derivatives.
4. Preview LG/W Lesson 26 and Stewart Section 2.9 and 3.8

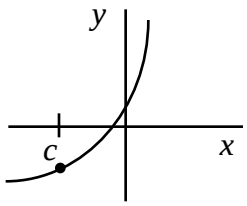
**AP Exam Sample Problem**

1. Let $f$ be a function with the following properties:

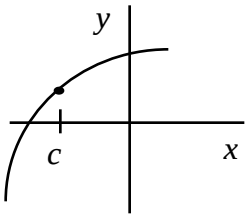
$f'(x) > 0$ for all real $x$; $f''(x) > 0$ for $x < c$; $f''(x) < 0$ for $x > c$.

Which of the following could be the graph of $f$?

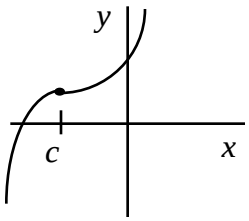
a)




b)




c)

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